Physics since AdS/CFT,
or how the universe keeps getting simpler but stranger.

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Introduction

This article attempts to summarize recent developments in spacetime physics. It's an exciting time. Ideas from several different realms are converging on a new model for how the world works. It's not just particles and forces any more. The world acts like a quantum computer.

Information theory is the new kid on the block. Physicists are making progress understanding black holes using ideas first conjured by communications engineers, and vice versa computer scientists are employing ideas from physics to optimize computer code. Complexity theory of emergent systems, a favorite of biologists, now is producing fruits in physics as well. Meantime, physicists are finding new connections between old ideas, with surprising implications. The goal of this article is to provide some background for these developments and to paint a plausible, but far from complete, picture of how the world works.

Two problems

Physicists are trying to solve two outstanding problems. (There are others, but two especially stand out.) First, what is the physical structure of space and time? Second, how can we reconcile general relativity and quantum mechanics, the two pillars of physics? It turns out those two problems are closely related.

Physical theory historically assumes that events occur on a pre-existing background of space and time. With Newton's laws, we can calculate the orbits of planets around stars as if they were projected on a coordinate system of meter sticks and clocks. Einstein's relativity theory says that meter sticks stretch or shrink depending on the concentration of mass in their vicinity, and clocks slow down when they accelerate, e.g. in a gravitational field. But the theory still assumes that clocks and meter sticks or comparable measuring tools exist. Only recently have physicists begun to tackle what the clocks and meter sticks are really made of, i.e. what is the structure of the "emptiness" out there between the galaxies.

The second problem, reconciling inconsistencies between quantum mechanics and general relativity, is more subtle. Quantum field theory (QFT), based on the principles of quantum mechanics, is the most accurate physical theory we have. It describes processes at the very smallest scales: why quarks collect in protons and neutrons to form atomic nuclei, why electrons are attracted to nuclei to form atoms, why atoms absorb and emit light at particular wavelengths, etc. The mathematical equations of QFT agree with experimental measurement out to one part in a thousand million million, the limits of our current capacity to calculate and measure such things. General relativity (GR) is right behind in the accuracy of its predictions, limited only by the fact that it deals with the very largest structures in the universe – neutron stars, black holes, galaxies, and the universe itself – where measurements get messier just because of the enormous scale. With over one hundred years of observations testing its predictions, general relativity has passed all tests with flying colors. The problem is that there's no mathematical theory (except maybe string theory, still under development) that includes both QFT and GR in its framework. We know there are circumstances where both theories should apply. For example, particles are produced at the event horizon of a black hole (the so-called Hawking radiation). QFT can describe the particle production. GR can describe the black hole and its horizon. But no single theory yet exists that describes both.

Enter quantum computation and information theory. Recent collaboration between computer scientists and the physics community has generated a lot of excitement, chipping away at these problems. That will be the purpose of the rest of this paper, to report new ideas relating the science of information to our traditional understanding of physics. First some background in what the information stuff is all about.

Information

What is information? In the field of communication, information has a precise definition first recognized in the last century by researchers including Claude Shannon at ATT Bell Labs and Charles Bennett at IBM Research. At its essence, information is ones and zeros, yes vs. no, heads vs. tails. Information is the response to a yes-no question, and it comes in bits, either 1 or 0. Is direct sunlight coming through your window right now? Yes or no? If yes, label that a 1. That's the relevant *bit* of information. Is there an electron in register AF10H24B of your computer memory? Yes or no? If no, label that a zero.

All information can be encoded in 1's and 0's, bits. It's as if nature plays an ongoing game of twenty questions. Is there a hydrogen atom at this particular location x, y, z at this particular time t? Yes or no. Is it moving at 10 m/sec? Is its spin up? Of course, we simplify our description of nature by consolidating information into standard measurements: what are the measured values of position, momentum, and spin of the electron. That saves a whole lot of yes-no entries into our data books. But in principle, we could describe the world in ones and zeros for what's happening at every location in space and time, including yes-no questions for all possible events at each location. And the spacetime locations themselves can be labeled with bits, also.

With this definition of information, it's convenient to encode messages as strings of ones and zeros – bit strings. Computers encode the alphabet in strings of eight bits. For example, to say "hi" send 01101000 01101001. If you want to be more enthusiastic, send "Hi!", 01001000 01101001 00100001. Even better, this convention allows you to process the message using standard mathematical operators (from the realm of linear algebra). For example, if you wanted to convert all the lower case letters "h" in a text to uppercase "H" you could scan the text for "h" (itself a bitwise operation) then carry out a matrix operation on the bit string, to flip that third bit in "h" from 1 to 0. That particular matrix operation is kind of messy (involving an 8×8 matrix) so we'll look at a simpler operation in a minute.

This idea of information caught the attention of physicists toward the end of the last century. What is physics anyway? We're trying to understand nature, what the world is made of and how it works. We're trying to read the book of nature. Wait – that rings a bell. We're trying to extract information about nature. And if information is, indeed, 1's and 0's, then we should be able to understand it as such. One thing led to another (that's the rest of the story in this paper) and pretty quickly physicists starting talking in ones and zeros. Suppose a photon, for example, flips the spin of an electron from spin down to spin up. Here's what that looks like in bit notation.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

where

$$down = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \;\; up = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ represents a photon acting on a spin down electron to flip the spin. More generally, matrices like $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ transform – stretch or shrink or rotate – vectors, represented e.g. by the original vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and the rotated vector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ in the equation above.

As in the example above, we model physical phenomena in mathematical equations. Math is a convenient (read that "indispensable") tool to represent nature. It not only describes what's going on, but its logical rules allow us to predict things we haven't yet seen. For some deep reason, nature herself follows those same logical rules. These particular mathematical operations come from standard linear algebra. Sal Khan's video, vector transformation, reviews the math behind this equation, if you'd like more explanation. Khan Academy, in general, is a great resource. Here's the link to Khan's Linear Algebra. Anyway, we'll explain the math as we go along.

Once they caught the bit bug, physicists found more and more meat in information theory. One of the foundations of information theory that attracted their attention is the conservation law for information: information is strictly conserved. No information is ever lost. It might become inaccessible (for example if your computer hard drive fails). But it's never lost. Nature keeps strict accounting, and every last bit of information is recorded in nature's books. Forever.

Here physicists are on familiar territory, and here we start to see cross-fertilization between the disciplines. Physics is built on the great conservation laws – conservation of energy, conservation of momentum, conservation of angular momentum, conservation of electric charge. (There are others as well, and all derive from a deeper principle, locality – but that takes us somewhat astray from present purposes.) From the fresh perspective of information theory, we can re-interpret those conservation laws in terms of information, maybe even make some new headway. Conservation of energy implies conservation of information about the state of a physical system, say a collection of atoms, over time. Conservation of momentum implies conservation of information about the position of an object over time.

Then there's unitarity. That's where the ideas really meld. Unitarity is a fancy term referring to probabilities. Flip a fair coin and there's 50% chance you'll get heads, 50% probability tails. But there's certainty, 100% probability, that you'll get one or the other. That's unitarity. Probabilities all have to add up to one, 100%. Given that any one of a number of things *can* happen there's certainty that one of those *will* happen. Roll a six-sided die and you know with certainty it will come up 1 or 2 or 3 or 4 or 5 or 6. Plant 100 tomato seeds and you know with certainty that none will sprout or 1 will sprout or 2 or 3 or 4 or … or all 100. And all that, from the information perspective, is conservation of info. Nature doesn't just throw away probabilities. It's not possible *not* to have tails as a possible outcome when you throw a fair coin. You don't lose the information about the state of the coin – it really does have a tails – when you toss the coin. And nature doesn't just add possible outcomes out of nowhere. There's no third possibility suddenly appears when you toss the coin. We'll see more of this when we get to quantum mechanics, shortly. And, as we'll see, it gets kind of contentious when we start to talk about black holes.

Classical circuits

Information can be stored; there are libraries filled with information and, of course, computer hard drives. It can also be processed. We can flip bits or extend bit strings or shrink them. The wonder is, to process information, we use information itself as the processor. We can use one bit of information to tell us what to do with another bit – flip it or send it to memory or just pass it along a circuit wire. As Alan Turing showed, information not only is the grist for computation but it also provides the instructions for the milling.

Here's a simple "full adder" circuit, for example.

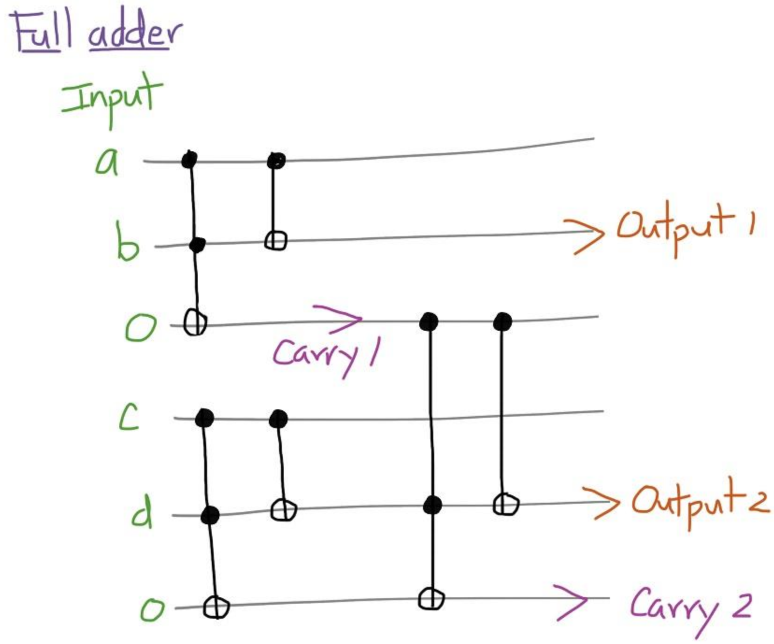


Figure 1. Full adder circuit. This circuit adds two-bit binary numbers. The result is a bit string of up to three bits. Wires 3 and 6 always start with an input bit of 0. ab and cd carry the input bit pairs to be added. To add one plus one, enter ab + cd = 01 + 01. The result is 010 (one plus one equals two, in binary notation). A couple other examples: 01 + 10 = 011 (one plus two equals three); 01 + 11 = 100 (one plus three equals four). The bit string output (far right of the diagram) reads from bottom to top. See example in the next Figure. Khan Academy [binary arithmetic] has more examples. This binary arithmetic is the basis of all computer operations and computer memory. The two gates in this circuit are the CNOT gate and the Toffoli gate (named after Tommaso Toffoli). Other gate combinations also could be used for the adder. See the gate figure on the next page for explanation how these two gates work.

The horizontal lines represent wires, which carry bits through time. They may be actual wires in a computer or radio waves carrying AM or FM signals or any of many other physical transmitters. Time runs left to right. We can parse the ticks of the clock as uniform intervals along the horizontal axis or in terms of the bit transformations, one after another, carried out in sequence by the logic gates along the circuit. Boxes represent single bit gates, acting only on the bit in that particular wire. An example is the NOT gate; it flips the value of the input. If input is 0, output is 1; if input is 1, output is 0. Vertical connecting wires represent two- or three- bit gates. The action of these gates on the target bit depends on the value of the bit(s) in the input wire(s). For example, a CNOT (controlled NOT) gate flips the target bit (wire to open dot) if the input (wire to solid dot) is a 1. If the input is 0 CNOT leaves the target at its original value. See the truth tables in Table 1., below.

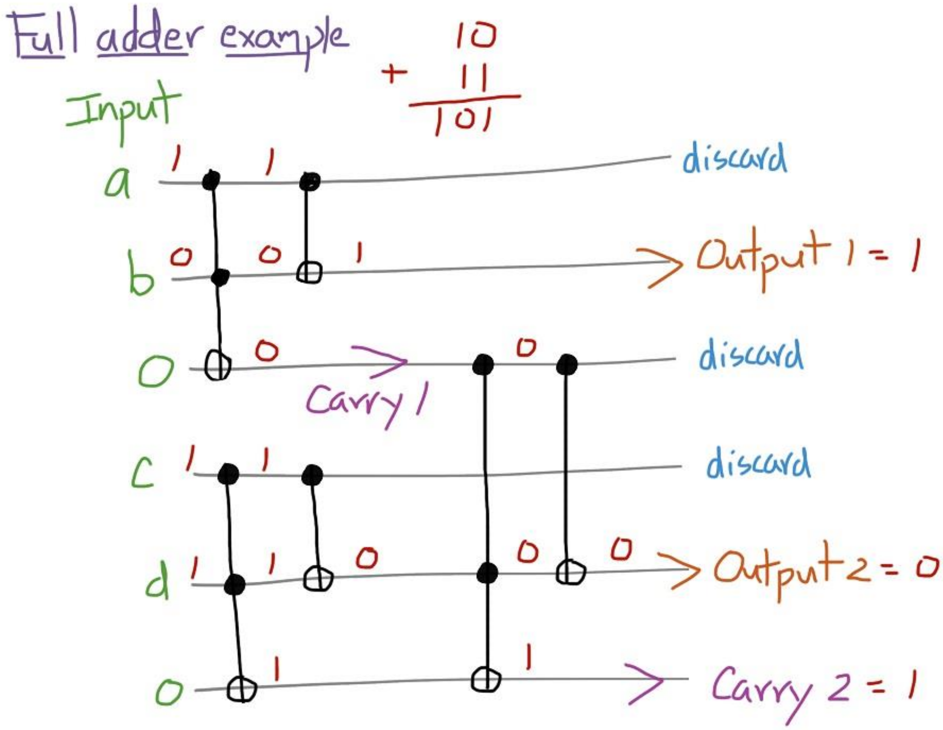


Figure 2. Example of a calculation with full adder. Two (10 in binary notation) plus three (binary 11) equals five. Red bits track bits along the wires as the various gates operate. Output in standard pencil-and-paper notation reads from bottom to top on the far right of the circuit output, 101. Carry 2 (a 1 in this example) becomes input **a** for the next adder in a larger circuit. Try out other sums – it's kind of fun to track bits through these circuits and watch the gates do their magic.

The full adder adds up to seven (111 in binary notation). If you link a series of full adders, with the carry value as input **a** to the next adder in the series, you can calculate any sum. And if you can add, then you can also subtract and multiply and divide. The full adder enables all basic whole-number arithmetic. That's most of digital computation right there.

It turns out you only need a handful of logic gates to build a circuit for any possible bit-wise computation. XOR (exclusive OR) and AND provide a universal set; the right combinations of just those two will build your computer. Just NAND by itself (not-AND) is universal. Apple could use circuits built from NAND to make the iPhone (but there are more efficient gate designs). Vice versa, a circuit with NAND gates can be decomposed into a circuit with combinations of other, simpler gates. See Table 1 on page 35 for a more complete set of bit logic gates.

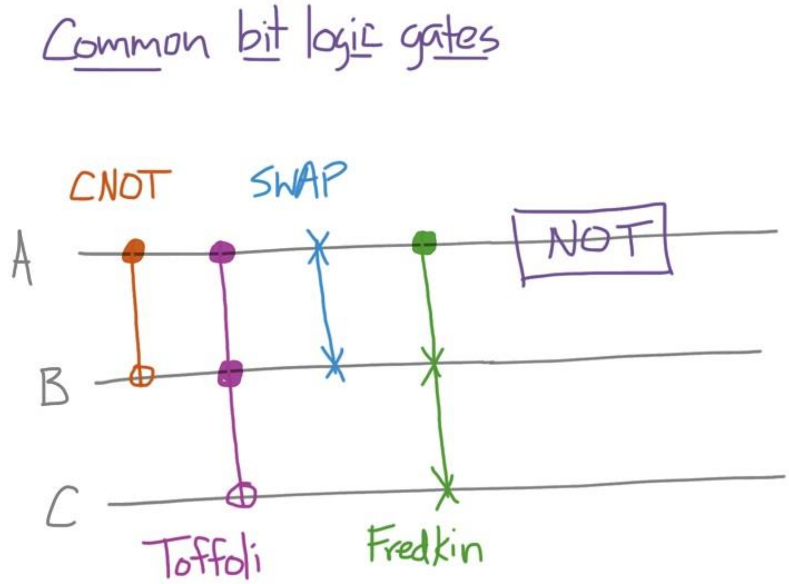


Figure 3. Some binary gates. Shown are gates useful to our purposes later on. Most circuits include AND, NAND, and OR gates, not shown. NOT is a single bit gate. It flips the bit in its wire from 0 to 1 or 1 to 0. CNOT and SWAP are two-bit gates. SWAP exchanges bits between two wires; CNOT flips the target bit (open circle) if the input (A in the Figure) equals 1, otherwise leaves the target bit unchanged if A is 0. Fredkin swaps B and C if A is 1, does nothing if A is 0. Toffoli flips C if both A and B are 1's, otherwise does nothing.

In circuit diagrams physicists saw new models for physics. Not only can you build circuits to run your calculations – they left that to the computer scientists, anyway – but maybe you can model the physics itself *in* the circuits. Two electrons exchange a photon that flips their spins – maybe that's a wire (electron moving in space and time) and a SWAP gate (photon exchanging spins). Further progress along those lines requires a dive into quantum computation.

Onward, then, to quantum circuits!

Quantum information and quantum circuits

So far we've been thinking classically. Bits are discrete, either 0 or 1. Quantum mechanics says a "bit" of information can be a 0 or a 1 or a little bit of both at the same time, a "qubit." Moreover, qubits can become entangled with other qubits in vast networks.

We need new symbols to contain these ideas. Up to now we've used the symbols 1 and 0 to represent bits. For qubits we'll use $|0\rangle$ and $|1\rangle$, the quantum symbols for vectors rather than numbers. Quantum mechanics models the world as state vectors in a vector space, and linear algebra is the language to describe it. We'll use electron spin as an example, but the ideas (state vectors and state space) apply to any property. There's a state space for every parameter, be it spin or position or momentum or color charge or any other characteristic of particles and fields.

Let's clarify those concepts, 'cause I'll use the terms when appropriate and without remembering they're not everyday vocabulary! (Susskind and Hrabovski, 2013, have a more thorough discussion of fields and state space, too.) A field is something that takes on a value at every point in space. Temperature is a field, technically a *scalar* field. You get a number (a.k.a. *scalar* value) when you measure the temperature at any point in a room. $20°\ C$ on the middle of the floor, $21.5°\ C$ at the northeast ceiling corner. The flow of water in a river is a *vector* field. The water has both a speed and a direction, 3 m/sec due west in the middle of the river, 1 m/sec east in an eddy along the bank. Electric and magnetic fields are the prime familiar examples of vector fields in physics.

As the name "quantum field theory" implies, physicists describe natural phenomena, especially particles and their interactions, in terms of fields. An electron, for example, is an excitation in the electron field. Think of electrons, as droplets of ocean spray excited by wave interactions in their ocean field. We typically visualize fields as expanses of wave trains on the broad ocean and particles as "wave packets," shorter wavelengths where the surface is disturbed by outside interference, like the wind, or where waves from different disturbances pile up. When we talk about particles, then, we're talking about field quanta, i.e. packets of field.

State space is like a catalog to keep track of the state of a system (a collection of particles). The state space of spin for a single electron is $|0\rangle$ and $|1\rangle$. When we measure its spin, the electron is either spin up (0) or spin down (1). (Notice I'm being careful to say these are the states when we *measure* the spin; we'll use numbers to show the results of a measurement. Shortly we'll see that, when we're not measuring them, electrons can be in a mixed state, neither up nor down. Generally speaking the vector notation and plain 1's and 0's are telling us the same thing.) With this notation, the (measured) state space for a system of two electrons is 00 (both up), 01 (first electron up, second electron down), 10 (first electron down, second up), and 11 (both down). That is, there are four possible configurations in the spin state space for two electrons.

For convenience, we can record the state space as a column vector (a list of possible states in column form). For example, here's the spin state space for a system of three electrons. Rows are the possible states, eight altogether. Notice that we're interested in the state of the *system* composed of the three electrons.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

This representation is convenient to help visualize what's going on: since we can record states as column vectors we can also *draw* those vectors. Back to the spin state space for one electron, for simplicity:

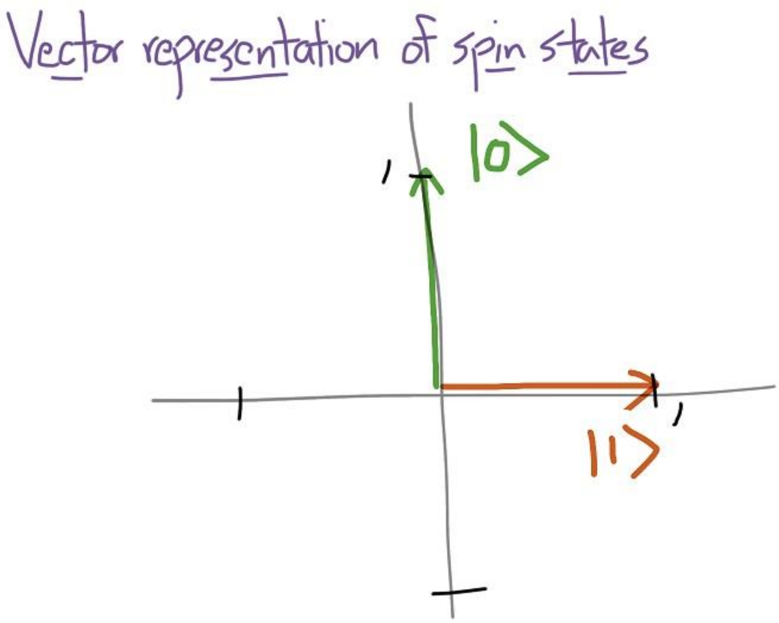


Figure 4. Vector representation of spins up ($|0\rangle$) and down ($|1\rangle$) on the coordinates representing the vector space of all possible spin states. Note a couple things. First, the coordinate axes are *not* the x, y axes we're used to in regular geometry. The axes here represent the direction of the electron's spin relative to some other, outside Cartesian (x, y, z) axes. For example, the vectors below might represent spin direction relative to the spatial z axis. Second, note also that the vector labels are arbitrary. By convention, we've chosen $|0\rangle$ as the spin up vector and $|1\rangle$ as spin down. Finally, note that these vectors are *orthogonal* (i.e. at right angles), *not* pointing opposite directions as we would expect in the regular world of ups and downs. This orthogonality in the vector representation follows the mathematical rules of linear algebra assuring that when we measure the spin of an electron it is either up or down – even though the real state of the electron, before any measurement, may be a mix of both!

We mentioned that measurements only read out 1's or 0's for the state of electron spin. Unobserved out in the world, electrons' spin axes point every which direction. Here's the full quantum expression for the state vector representing electron spin.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

The brackets tell you we're dealing with vectors. (Mathematicians have a whole bunch of different ways of representing vectors. This is the standard vector representation in quantum mechanics; there are some technicalities, for instance if we reverse the direction of the brackets we get a different kind of vector, but we won't need those subtleties.) $|\psi\rangle$ (Greek letter psi) is the conventional symbol for a state vector. α (Greek letter alpha) is the amplitude (component) of the electron's spin along the $|0\rangle$ direction, i.e. how much the spin axis is tilted upward. β is the amplitude of spin in the $|1\rangle$ direction, i.e. how much the spin axis is tilted downward.

One of the requirements of the quantum conventions is that $\alpha^2 + \beta^2 = 1$. We've seen this before. It's unitarity. We are requiring that the spin of the electron is pointing in *some* direction and that the magnitude of the spin always equals one. It's an inherent, invariant property of the electron, part of what makes an electron an electron.

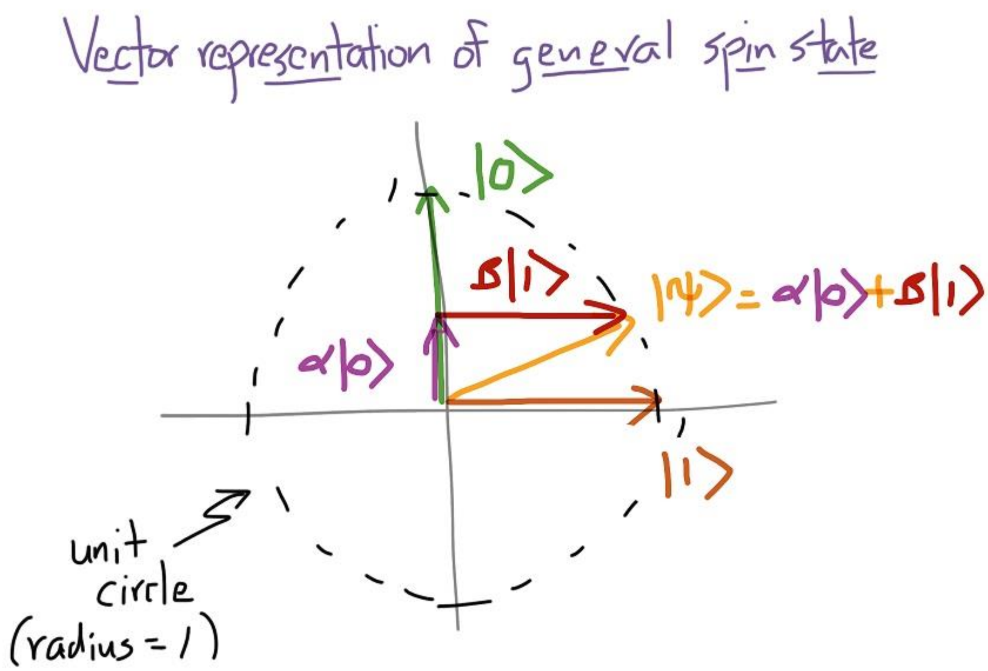


Figure 5. Vector representation of a general state vector, $|\psi\rangle$, showing its component vectors $\alpha|0\rangle$ and $\beta|1\rangle$. In this case, $|\psi\rangle$ is built from a proportion α of $|0\rangle$ and proportion β of $|1\rangle$. Note that $|\psi\rangle$, like $|0\rangle$ and $|1\rangle$, is one unit in length. This is the requirement of unitarity, assuring that calculations always give probability = 1 when you add all possible vector components for a particular state.

There are some subtleties here. Electrons aren't really like little spinning tops, but the real physical property of spin can be conveniently described on those terms. And there's nothing magical about the Greek letters. They're just handy when you run out of the good ol' Latin alphabet. a's and b's and p's and q's have already been taken for other purposes.

Back now to circuitry. Turns out we can understand a lot of basic quantum mechanics based on circuits, and we're most interested in circuits anyway.

Wires in a quantum circuit, instead of ones and zeros, are qubits. We represent them in vector notation, e.g. $|\psi\rangle$. The gates in a quantum circuit are vector operators, unitary matrices in the mathematical formalism. We've already seen an example in our classical circuit.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ flips the spin of an electron from down to up. That's a perfectly good quantum operation, by the way, assuming the electron is in a pure down state to start with, spin axis pointed straight along the down axis. Unitary means what you suspect. Unitary operators (matrices) preserve the probabilities, so that circuits (and the world) never produce more states than they started with.

One of the joys (or headaches) of quantum circuits is that any unitary matrix qualifies as an operator and, therefore, a gate. Just as in classical circuits, though, you only need a few kinds of gates to build any conceivable quantum circuit. See Table 2 on page 37 for a list of qubit logic gates.

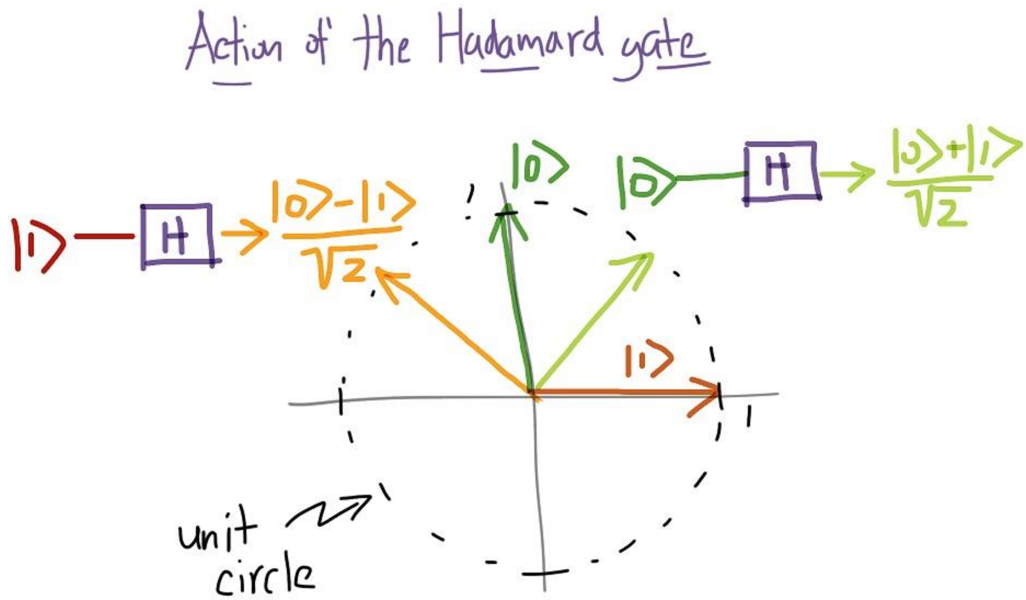


Figure 6. Action of the Hadamard gate on qubits $|0\rangle$ and $|1\rangle$. Note that Hadamard results in a mixed state; the original qubit is transformed into a mix of $|0\rangle$ and $|1\rangle$. In terms of the general state $|\psi\rangle$, Hadamard resets the values of α to $\frac{1}{\sqrt{2}}$ and $\beta \rightarrow \frac{1}{\sqrt{2}}$ or $\frac{-1}{\sqrt{2}}$. Note that, as required by unitarity, $\alpha^2 + \beta^2$ still $= 1$ after the transformation.

Now we can start to do some magic or, rather, replicate some of the magic that Nature performs. First conjuring is entanglement. Whenever two particles bump into each other, their state vectors become entangled. As a result, even after the two particles are separated in space and time, you can get information about the state of one particle by measuring the state of the other. Now, particles are always bumping into each other. (More properly, the fields that carry particle properties are always interacting.) Or, 'way back in the beginning, at the Big Bang origin of the universe, all the fields were jam-packed squished and all interacting, so the whole shebang is entangled. Extracting information *here* can give us bits of information about conditions out *there* across the universe and maybe even inside black holes.

We have all the circuit tools we need to generate entangled pairs of qubits. Just what is entanglement? It's easiest to see it in the state vectors. For example, here's an entangled spin state for a pair of electrons.

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Before we measure either electron, we don't know its spin. All we know is the overall state of the system represented in the formula above; we prepared the electrons in that. There's a 50% chance both electrons are spin up, 50% chance they're both spin down. But if we measure the first electron (labeled green) and find it's spin is up, then we also find, with 100% certainty, the second electron (labeled by magenta) has spin up. And if we measure the first electron and find it's spin is down, then we also find the second electron has spin down. That's entanglement. We can determine the state of one electron by measuring the other.

To build such an entangled state, all you need is two qubits for input, then a Hadamard gate followed by a CNOT. Presto! You've got an entangled pair, a so-called Bell pair (named after John Bell, who studied the marvelous properties of such creatures and, with them, proved that quantum mechanics cannot be built from standard classical logic).

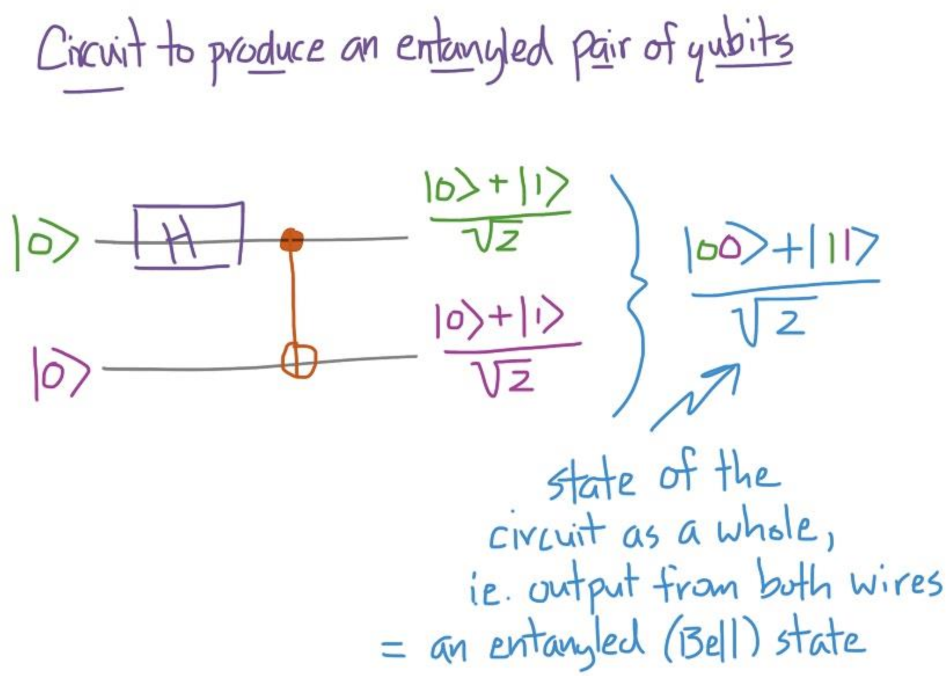


Figure 7. Circuit to prepare an entangled pair of qubits. Input qubits in this example are both $|0\rangle$. A Hadamard gate produces a mixed state in the top qubit, and a CNOT transforms the lower qubit based on that mixed state. Note that CNOT acts on the bottom $|0\rangle$ twice, first with the $\frac{|0\rangle}{\sqrt{2}}$ control and then with $\frac{|1\rangle}{\sqrt{2}}$ to produce the mixed state in the bottom wire. The output

superposition of both wires is an entangled state referred to as B_{00} , the Bell state produced when both inputs are $|0\rangle$. See if you can figure out the other Bell states, B_{01}, B_{10}, and B_{11}.

Quantum mechanics won't allow us to measure all the details of a full state, ψ. When we measure a particle's spin, for example, we must choose the orientation of our measuring device, say along the z axis. Electrons passing through that device may have spin oriented any which way, but all we can detect is their component of spin along z. For any particular electron, all that the detector can tell us is "spin up" or "spin down." If we measure lots of electrons that were all prepared in the same state, then we can count how many spin up's we measure and how many spin down. ("All prepared in the same state" is key here.) Those counts give us α^2 and β^2 for the state ψ in which the electrons were prepared.

With entanglement we can do wonders. Entanglement enables circuits to send two classical bits of information using just a single qubit. This "superdense coding" allows a sender, Alice, to send twice as much information to a receiver, Bob, at the same cost of computation.
Even more, entanglement allows teleportation. It's not yet (and probably never will be, because of practical limitations) the "beam me up, Scotty" teleportation of Star Trek. But it has already been accomplished in a variety of physical systems using quantum circuitry. Alice can teleport a Bell state to Bob.

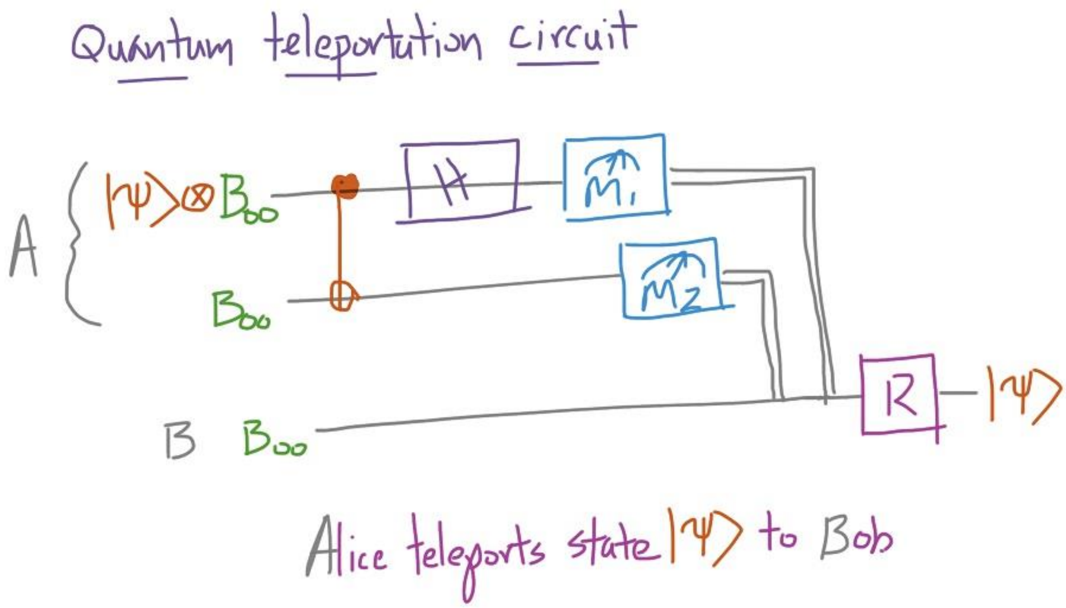


Figure 8. Teleportation circuit. Alice, A, processes two qubits to teleport the state $|\psi\rangle$ to Bob, B. One of Alice's inputs is the tensor product of $|\psi\rangle$ with the Bell state, B_{00}. We'll skip the details of tensor products; think of this product as in standard encryption – the product from multiplying the message times a prime number, the key. Later, if Bob knows that prime key, he can divide the product to extract the message. This teleportation circuit is a bit more complicated, but that's the essence. Alice entangles the tensor product with a second Bell qubit. Then she measures the qubits in both wires. As we've seen, measurement reads out a classical bit; that's what the double lines represent – bits rather than qubits. Alice sends those measurements to Bob over a standard circuit (hence there can be no faster-than-light teleportation). The bit pair, $00, 01, 10$, or 11 that Alice sends is the key that Bob uses to replicate $|\psi\rangle$ from his own Bell state. The replicator is one of the rotation gates, R. Bob rotates B_{00} around the *X* axis if he receives 01, around *Z* if he receives 10, and around *X* then *Z* if he receives 11. If Alice sends 00, then B_{00} itself is $|\psi\rangle$.

We won't go into the details how these circuits work, but if you'd like to look behind the curtains in this magic show, check out Nielsen and Chuang, 2008, or see Nielsen's YouTube lectures on the subject (Nielsen, YouTube 2014).

A final comment on these circuits and how they help to illustrate the mechanisms of quantum mechanics. We've used vectors in two-dimensional space as examples. As we've already mentioned, though, nature is a jumble of fields – electron fields and photon fields and the weak and strong fields and the Higgs field and more. To fully describe the state of any particle, we have to include all the appropriate fields. But then, maybe that's not so bad. We might not have to imagine vectors in six dimensions to represent six fields. Maybe the circuits, with appropriate gates, can simplify things. This is speculation, but maybe circuits can help us understand complicated field interactions in terms of information processing. Feynman diagrams, the standard representation of field interactions, sure look like quantum circuits, and their components behave like wires and gates. We'll consider these notions in more detail later.

Dual spaces

If the universe is entangled – connections all over the place and interactions mediated by gate-like operators – maybe we can think of it as a computer. Maybe it *is* a computer, and maybe it can be understood in terms of its circuits. That's outlandish. On the other hand, theorists have proposed connections that sure seem like circuits. Very good theories supported by rigorous math. We'll look at some of those theories in this section.

First up is Jacob Bekenstein. In a paper published in 1973 (Bekenstein, 1973), Bekenstein showed that the information in a black hole is distributed on its boundary. That was a shocker, for a number of reasons. First of all, stuff falling into a black hole – stars and refrigerators and copies of the Oxford English Dictionary – presumably fall through the horizon into the interior volume of the black hole. But the information is not distributed into the interior volume. The information is distributed randomly across the horizon. In technical terms, the area of the event horizon of a black hole measures its entropy.

OK. So what? Well, information theory says entropy is missing information. Or rather it's information to which we don't have access. All the information from that dictionary still exists (by information conservation) but it's scrambled across the horizon. Ones and zeros all mixed up, each bit covering about a Planck area (really really tiny) on the horizon. Presumably, if we could unscramble it, that information on the horizon could tell us about the interior.

That's extraordinarily puzzling. How can all the information encoding a three dimensional space be recorded on two dimensions? If that same reasoning extends also to spacetime outside of black holes, it's as if the world is a hologram. (Susskind, 1994; Bekenstein, 2007) Just as a hologram is a 3-D laser projection from 2-D film, this notion implies that our 3-D world may be a projection from a 2-D . . . what? (String theorists would say it's a projection from a "brane," but that's a topic for another time.)

Great physics follows from outlandish notions. Maxwell's equations from invisible fields. Relativity from the impossible idea that the speed of light is the same for all observers moving at any speed. In 1997 Juan Maldacena took on the outlandish notion of holography and discovered a solid mathematical connection between interiors and boundaries (Maldacena, 1997). In what has become the most cited paper in all physics, he established that the mathematics of quantum field theory in *D* dimensions also describes a system including gravity in (*D+1*) dimensions (at least in certain model geometries). All the information that is required to describe a three-dimensional space is contained on the two-dimensional boundary of that space. Maldacena's model includes a volume, the "bulk," which has the mathematical structure of anti-de-Sitter space. The surface enclosing the bulk, the "boundary," has the mathematical structure of conformal field theory; hence the acronym AdS/CFT (Anti-de-Sitter space / Conformal Field Theory). And, most importantly, the mathematics of the bulk is dual to math on the boundary. That is, you can describe the same physics using either of the two maths. Your choice. Whichever makes the calculations easier.

It's the duality in Maldacena's theory that is so useful. Using the relevant mathematical equations, gravitational problems that are intractable in *D* dimensions might be simple to solve in the *D*-1 mathematics of field theory. Vice versa, beastly problems in field theory might resolve easily using the dual mathematics of general relativity. AdS/CFT provides the appropriate tool kit for various jobs in its purview, nicely calibrated wrench sets when all you might have otherwise are pliers.

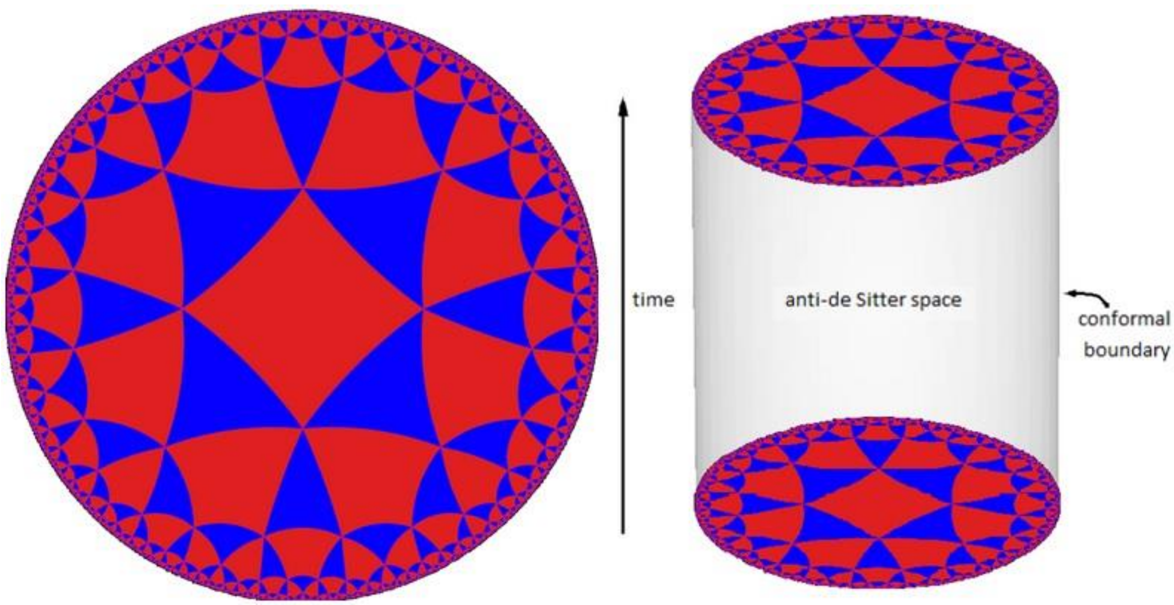


Figure 9. Anti-de-Sitter space with conformal field theory boundary. In left-hand image, AdS is represented by a hyperbolic disk, referred to as the bulk. Each triangle has the same area; imagine the edge curving off to infinitely far away, so distant triangles look smaller. The boundary is the edge of the disk. In this representation, the mathematics of quantum field theory on the 1-dimensional boundary derive the same results as 2-D spacetime math (e.g. general relativity) including gravity in the bulk. Right-hand image shows AdS evolving through time. Image fromWiki media.

In Maldacena's original paper, the bulk is 5-dimensional and the boundary 4-D. Our actual 4-D bulk spacetime universe has a different geometry. While AdS has a negative hyperbolic curvature (like a saddle shape) where light rays that start out parallel eventually diverge, observational and theoretical evidence show that the geometry of our universe is flat (parallel light rays continue forever on parallel paths). It remains to be seen whether we can find a similar duality for our flat spatial geometry and its boundary. Meantime, AdS/CFT has stimulated enormous progress in many fields, including black hole physics, particle physics, and condensed matter physics (the study of solid materials and including, e.g., superconductivity). In those disciplines the "toy model" mathematics of AdS/CFT have produced extraordinary real-world discoveries. And, more to our purposes, AdS/CFT has enabled major advances in understanding the structure of our own spacetime.

Connections between bulk and boundary

Next question: if information in the bulk is dual to information on the boundary (a fancy way of saying AdS/CFT applies, which is a fancy way of saying holography applies, i.e. all the information in the interior is recorded on the surface), then what is it that provides the connection? How is the bulk connected to the boundary? Three recent developments, likely all connected (or entangled), mark considerable headway toward an answer.

First is work by Mark van Raamsdonk (van Raamsdonk, 2010), building on ideas introduced by Shinsei Ryu and Tadashi Takayanagi (Ryu and Takayanagi, 2006). It's pretty straightforward, given our discussion of quantum entanglement. The bulk is entangled with the boundary.

Here's the argument. Ryu and Takayanagi showed that the information entropy on a patch of boundary in AdS/CFT is proportional to the minimal geodesic in the bulk connecting the endpoints of the boundary (Ryu and Takayanag, 2006). Swell. Come again? Well, it's as if information on a patch of boundary is telling you what's in the volume right underneath. Packets of information on the cap of boundary tell you what's in the bottle of bulk underneath. Kind of like if you're stocking shelves in the grocery and receive a mixed carton of jams. Reading the label on the cap tells you what's inside that particular bottle in the bulk (carton) jam space. Or, alternatively, you could open the bottle, taste the jam, and determine what must be written on the cap label.

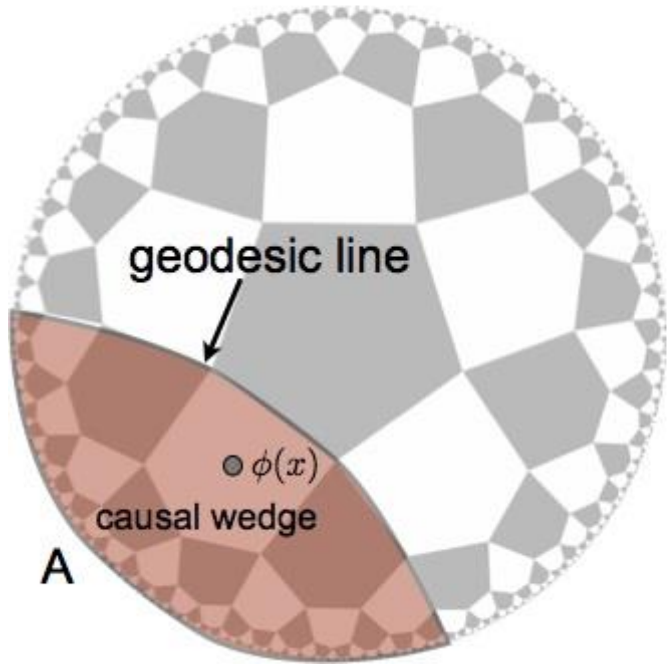


Figure 10. Geodesic sector for information on the boundary in AdS/CFT. Image shows a "causally connected" sector AdS/CFT enclosed by an arc of boundary and a geodesic – the shortest spacetime path connecting endpoints of the arc through the bulk. ϕ is a bulk operator, i.e. a mathematical procedure that can change the state of a local qubit; think computer circuit gate. Causally connected means signals from such a sector of the bulk can reach the boundary of that sector and not elsewhere. Information in that sector is entangled with its boundary. Image credit Beni Yoshida.

Building on this idea, van Raamsdonk went on to show that spacetime in the bulk is *created* by the entanglement. According to the mathematics of AdS/CFT, if you disrupt entanglement in the bulk, you decrease the volume of its spacetime. Spacetime originates from entanglement. It's as if you walk into a room filled with cobwebs. As you cut through the cobwebs, they collapse back to the walls and ceiling. The interconnected fabric we call spacetime disappears.

Another, complementary model is under development by Brian Swingle at Harvard and by John Preskill's group at Caltech. This model returns us to our main topic of quantum computing. The idea is that the bulk of AdS is a tensor network, a quantum circuit with gates linked by wires, and that the bulk operates as a giant quantum error correcting code. (See Ouellette, 2015, for an overview.)

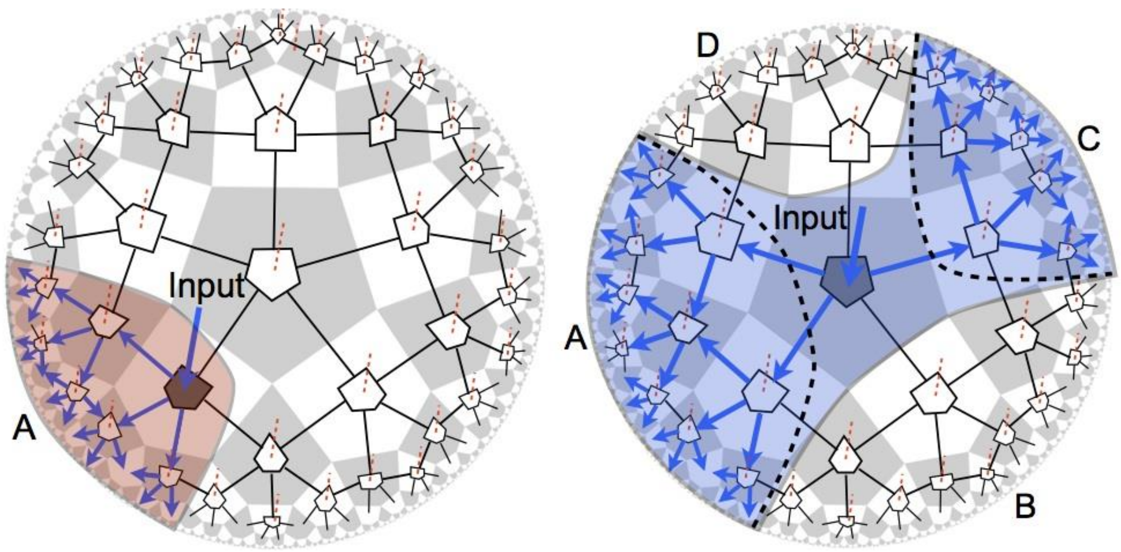


Figure 11. Tensor network. Pentagons represent tensors (think quantum circuit gates) each processing inputs coming from center of the bulk and distributing three output qubits to the periphery. Shaded regions show entanglement resulting from this particular tensor network. Other networks can have different tensor composition. Image credit Beni Yoshida.

Tensor networks help solve a problem inherent in Figure 10. Suppose you have a bit of information or an operator smack in the center of AdS. You can include that bit in the causal wedge of any of three or more boundary arcs, each of those encoding different information.

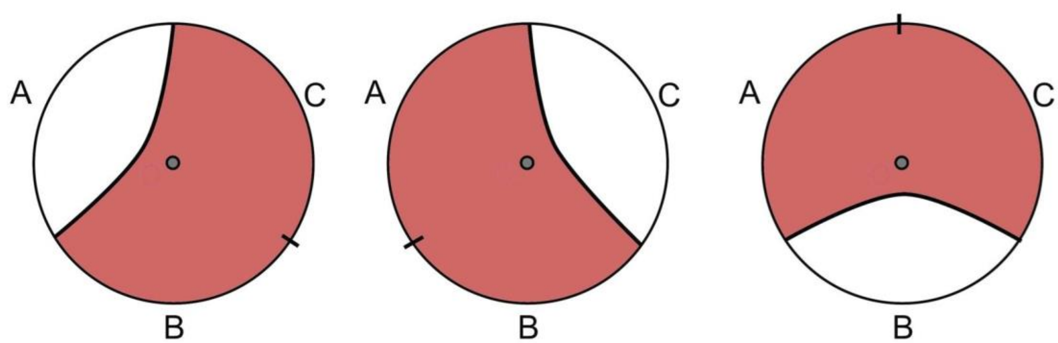


Figure 12. Three bulk causal sectors, in red, defined by three boundary arcs BC, AB, and AC. An operator at the center of the bulk is included in all three sectors.

But the central bit is the same, so you should be able to recover its information on any of the three arcs. How to? Swingle and Preskill et al propose that entanglement coded by the tensor network distributes information throughout the bulk so that it appears the same all around the boundary (Swingle, 2009; Pastawski et al, 2015). Entanglement distributes information so that it's no longer isolated; it's all over. Kind of as if God injected a little drop of blue right in the center of a can of red paint then stirred the paint so that bit of blue got distributed everywhere, including into the touch of color sample brushed on the outside of the can. Such "error correcting codes" use entanglement to distribute information. That distribution helps prevent information loss, and error correcting codes are being incorporated into real algorithms in real quantum computers.

We've reached our goal, to explain the structure of spacetime. It's entanglement. At least that's a plausible model, and supported by the math. But it's based on that toy model, AdS/CFT. What is it, exactly, that's entangled? And does the same reasoning apply in our real universe, not just AdS/CFT?

ER=EPR

Enter ER = EPR. The acronym derives from the authors' initials on two of Einstein's papers, both published in 1935 a few months apart (Einstein et al, 1935). (There's interesting history here. The two papers, when first published, had no obvious relation, and Einstein himself never accepted the underlying quantum mechanics, entanglement especially, for which the papers have come to play a central role!)

Einstein and Nathan Rosen (ER) discovered wormholes. In an effort to avoid singularities in the Schwarszchild solutions to general relativity (which predict black holes), Einstein and Rosen proposed tunnels through spacetime to bypass them. (Singularities are conditions, e.g. at the center of a black hole, where the equations of GR give infinities as solutions, as in dividing by zero. Infinities cannot be interpreted as actual physical conditions.)

Einstein, Rosen, and Boris Podolsky four months later formulated the "EPR paradox." They devised a thought-experiment designed to prove that quantum mechanics could not be a complete theory of nature. They argued that a measurement performed on one particle separated by a large distance from the other particle in an entangled pair would immediately determine the state of the other particle. That, they said, violates the principle of relativity, that no information can travel faster than the speed of light. Turns out, as we've seen in our discussion of entanglement, that argument doesn't obtain; it just shows that quantum mechanics does *not* obey the rules of classical physics. On the other hand, reading between the lines, it suggests that two entangled particles are physically connected – by a wormhole.

That's ER=EPR. The conjecture by Maldacena and Susskind (Maldacena and Susskind, 2013) is that entangled particles are connected through wormholes. If all the universe is entangled – all the particles here there and everywhere – then the physical structure of spacetime is a cobweb of wormholes.

Emergent gravity

All done. We've got a model for spacetime. It's a wormhole web of entangled fields, and we can describe it with quantum circuitry.

Well . . . not quite done. We've been mathematizing in AdS/CFT, not in our universe. We need a theory that works in the universe as we know it. Moreover, we need to figure out where gravity comes from. General relativity describes gravity as spacetime curvature. Space and time stretch and bend from one region to another. It's geometry. The presence of mass – or more accurately the presence of energy in any of its many forms – distorts the geometry of space and time. The trajectories of objects moving in that geometry, then, follow curved paths. That's what we call gravity. How do we get spacetime curvature out of an entangled web of wormholes or out of a tensor network?

Erik Verlinde has applied some of this AdS/CFT thinking to models in the real universe, with suggestions for a solution to the gravity problem (Verlinde, 2016). He uses the bulk / boundary model, but Verlinde's bulk is our observable universe and the boundary is the cosmic horizon, the "edge" of the universe beyond which we cannot see. Looking out into an expanding universe such as ours, one that is accelerating, you eventually probe a distance at which galaxies are receding faster than light. Light from them can never reach us. By Verlinde's string theory calculations, gravity is a thermodynamic property akin to osmotic pressure in liquids. Mass / energy is attracted into clumps because it dilutes the spacetime "fluid" in the vicinity, like salt dilutes the water in which it is dissolved. More fluid flows in from the bulk, just as water flows by osmosis, dragging more mass / energy along with it. Verlinde's model is more sophisticated than that, including various degrees of entanglement within the bulk and outside the horizon, but that's a nutshell summary. What's new here is gravity as an emergent property. Gravity isn't a separate force. It arises secondarily from other properties of spacetime.

We can build on that, perhaps. Here's the grand finale. Drum roll, please.

Spacetime as a Feynman circuit

Physicists have developed a model of the world, on solid mathematical foundation, with these essential ingredients: Information is key. Quantum circuits, which process information, mimic physical processes. Circuits also provide plausible mathematical structure for spacetime. Can we build spacetime out of circuits that are based on known physics and in which gravity is an emergent property? What follows is a whole lot of speculation, but it's plausible.

First requirement: information has a physical basis (Landauer, 1991). If spacetime is circuitry, there must be some physical system that stores and processes that information. What could that be? Well, Feynman diagrams sure look like circuits

Richard Feynman famously invented his diagrams to facilitate calculations in quantum field theory. Straight lines on a spacetime plot represent particles moving through space and time. Wiggly lines represent the forces by which particles interact with each other. Vertices, where lines intersect, represent scattering processes in which particles exchange properties such as momentum, color, flavor, spin, etc. Each line and vertex is associated with a mathematical formula.

An early motivation for the link between Feynman diagrams and spacetime is found in the classic text, *Gravitation* (MTW), (Misner et al, 1973) which presents Einstein's model of spacetime as a web of events. There's no background on which the events occur. Events themselves build the structure of spacetime.

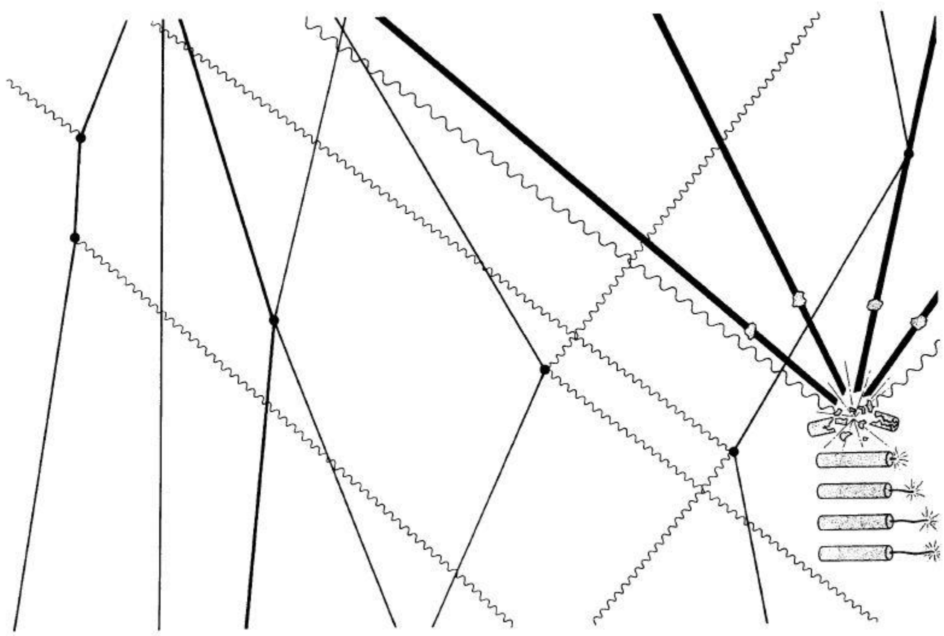


Figure 13. Spacetime as a web of events recorded on a Feynman diagram. Vertical axis is time, horizontal axis is position. Straight lines represent particles, wavy lines represent photons (light). A fire cracker goes off at bottom right; dark lines represent fragments from the explosion. From Figure 1.2. in Misner et al.

In-coming and outgoing particles (the quantum states of fields) are like wires in a circuit; bosons (interactions) are like gates. Such an extrapolation may not be too far-fetched. The mathematical formalism embodied by Feynman diagrams includes the same elements as in circuit gates (see e.g. Nielsen and Chuang, 2017). Wires are quantum states; so are particles. The same unitary matrices that function as circuit gates also describe particle interactions.

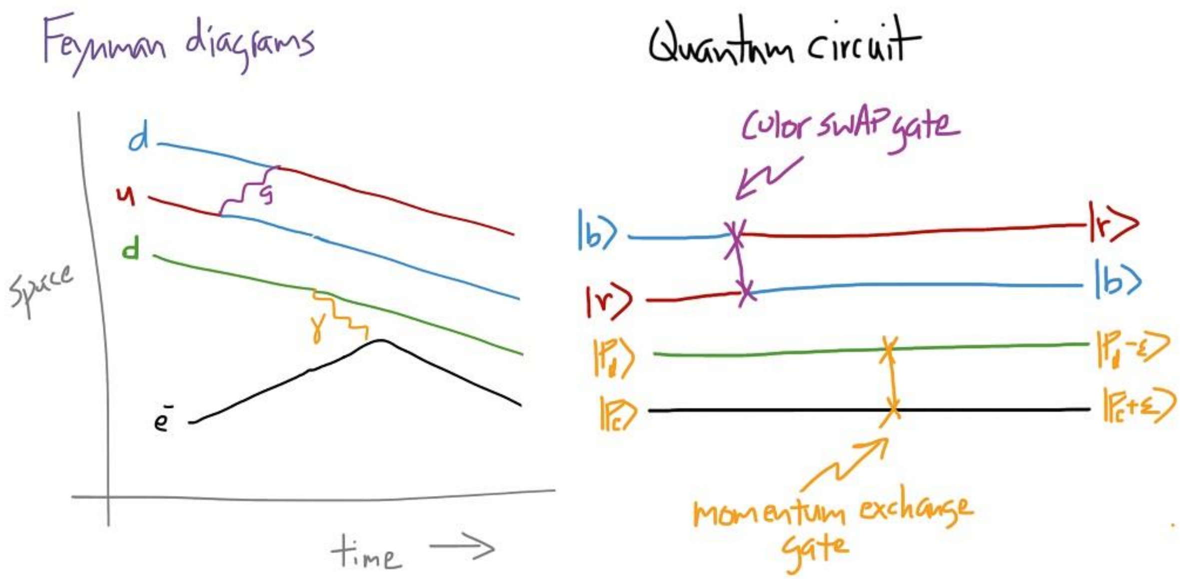


Figure 14. Feynman diagrams as quantum circuits. The Feynman diagram below is rotated from the standard representation, where time is the vertical axis and spatial displacement runs along the horizontal axis. The world is a network of events, i.e. field interactions. Perhaps it can be modeled as a quantum circuit; fields are the wires and interactions are mediated by logic gates. In the diagrams below two down quarks exchange color (on the left), as if by a color-SWAP gate (diagram on the right), and a photon exchanges momentum with a down quark (left) as if through a momentum-SWAP (right). Other gate combinations might work, also.

A photon might be a swap gate exchanging spins when two electrons scatter, or the exchange of a red / anti-blue gluon might be a rotation gate in "color space" exchanging color states of a red quark and a blue quark. And the weak interaction might look like this in quantum circuitry.

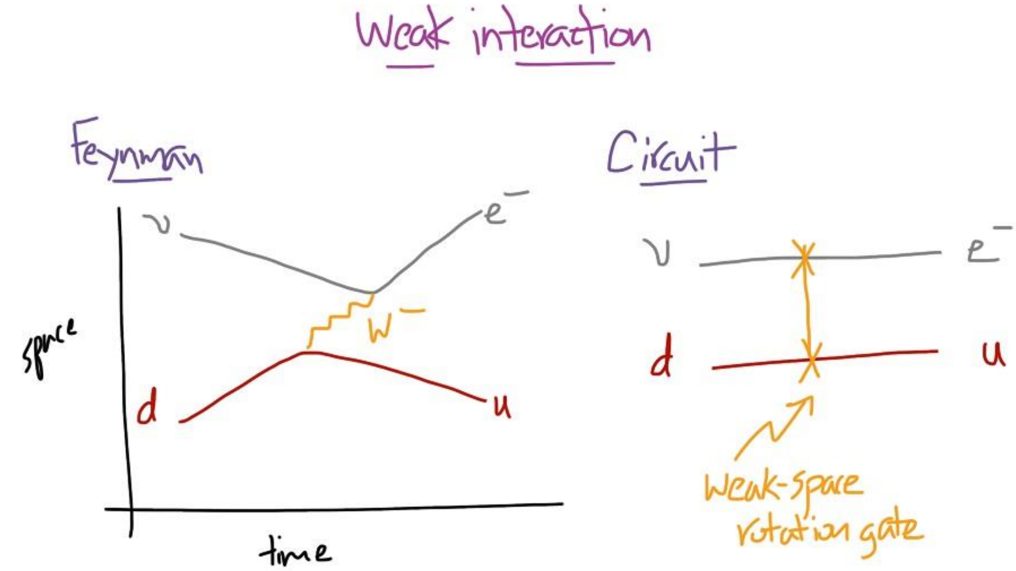


Figure 15. The weak interaction as a quantum circuit. Feynman diagram on the left shows transformation of a down quark into an up quark (a change in particle “flavor”), mediated by a W^- vector boson. A neutrino participates in the reaction, and an electron emerges as a product. The circuit gate is speculative. The criteria for such gates are the same as for the scattering matrices in Feynman diagrams. We can think of the W^- as a mathematical operator, the scattering matrix, transforming a $\nu + d$ field state into $u + e^-$. See Figure 16 for a state space representation.

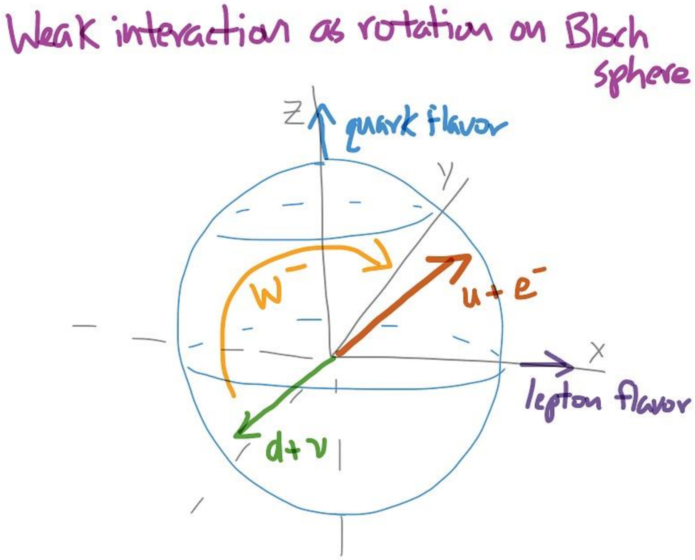
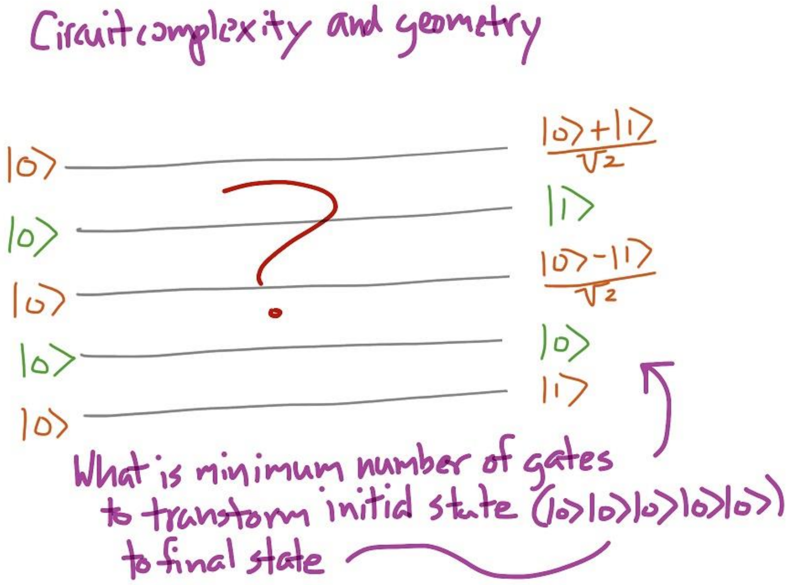


Figure 16. Another representation of the weak interaction as rotation in three-dimensional "flavor space." W^- rotates the $\nu + d$ state vector into $u + e^-$. Such a vector transformation can be represented by a combination of rotation gates in a circuit.

Crazy stuff, and speculative. We're looking for mathematical tools to describe nature. This might (or might not) be one. Are quantum logic gates sufficient to describe all Feynman diagrams? The rules of quantum field theory require unitary operators to carry out particle interactions seen in the Feynman diagrams. The rules of quantum circuits require unitary operators as gates. Coincidence? It seems plausible that the gates' capacity to change basis (i.e. the vectors we use in coordinate systems), change phase, and to rotate state vectors in state space might suffice to model all the particle interactions.

Emergent gravity

Where is gravity in this picture? Nielsen et al discovered that the complexity of a quantum circuit can be described in terms of geometry (Nielsen et al, 2006). Measure how many gates it takes to get from the initial state to a new equilibrium state. The minimum number of gates, it turns out, is analogous to a geodesic in good ol' Riemann geometry, the geometry of general relativity, so circuit complexity is a measure of geometric curvature (Brown et al, 2017). Roughly speaking, denser circuits with more gates – or more particles and interactions in a system – have greater curvature. In this picture, gravity is an emergent property of the circuit.



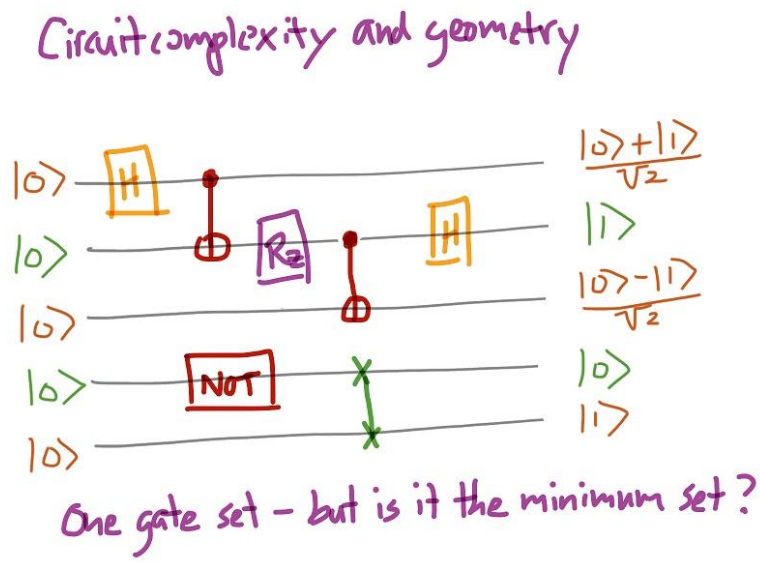


Figure 17. Complexity of a circuit can be measured as the minimum number of gates required to reach a given final state from an initial, simpler state. That gate configuration has an associated geometry and curvature.

Leonard Susskind's group at Stanford proposed gravitational dynamics for such systems (Brown et al, 2015). Start with a stable circuit, in equilibrium. Then add another gate to the circuit. The gate changes the states of the wires (particles) on which it acts. Information in those wires changes the output from the gates on downstream in the circuit. For example, if a CNOT or

Toffoli gate receives a $|0\rangle$ instead of $|1\rangle$ input, it no longer flips the bit in its target. The changes ripple through the circuit until it reaches a new equilibrium, like gravitational waves rippling through spacetime.

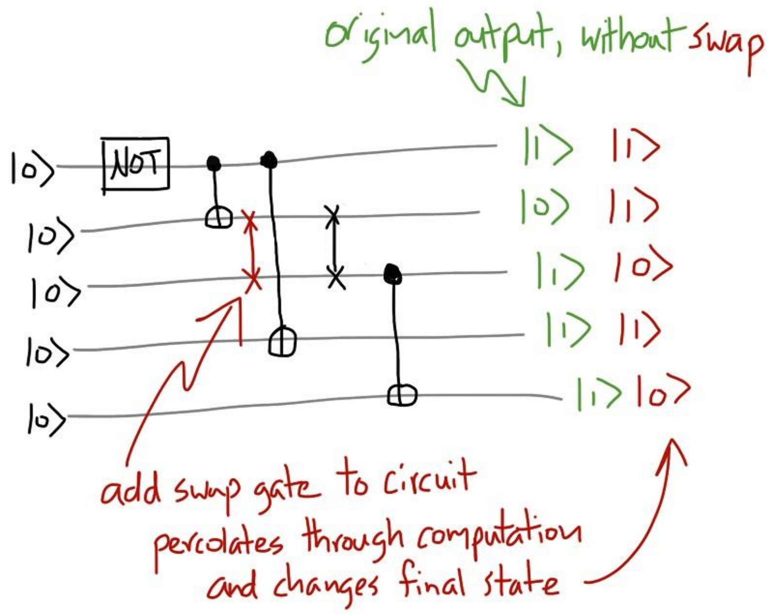


Figure 18. Inserting a new gate into a quantum circuit creates a ripple through the state space and results in a different final state. Such dynamics might model gravitational interactions.

What about the vacuum, the void out there between the galaxies? The stuff remaining in a vacuum chamber when there are no particles? How can the vacuum be represented as circuitry? Maybe as wires (fields) without gates. Every now and again a wire splits, mediated by one of Feynman's branching gates (Feynman, 1999), then merges – virtual pairs out of the vacuum. Unitarity, conservation of information with time reversal symmetry, requires that those branching gates return the system immediately to its vacuum state.

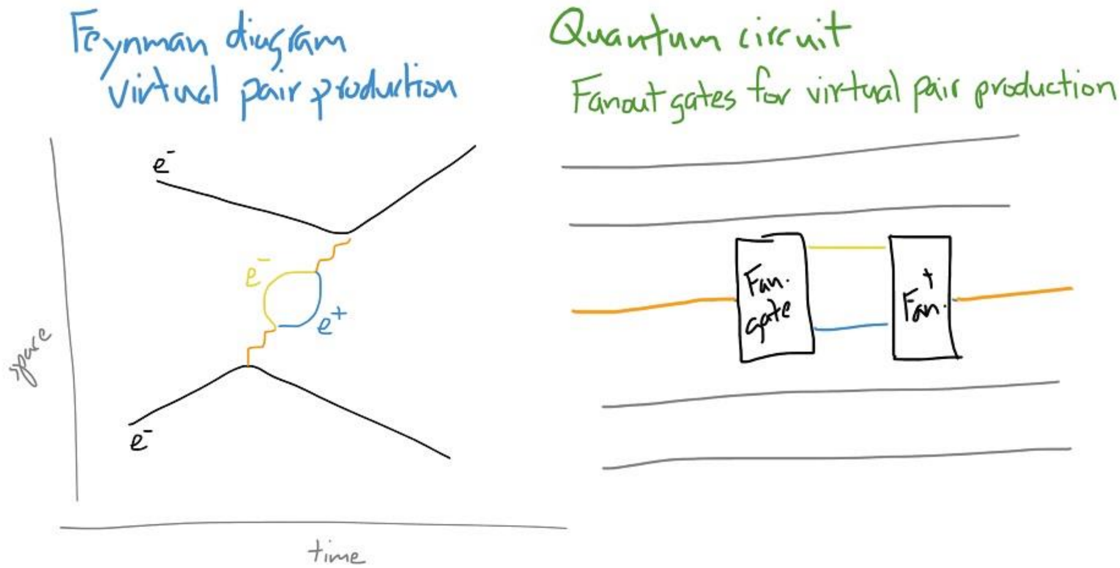


Figure 19. Virtual pair production out of the vacuum modeled with FANOUT gates (extrapolated from Feynman, 1999). $FAN^{\dagger}$ is the inverse of the FANOUT gate, a.k.a. a FANIN. i.e. it reverses the action of FANOUT.

Can circuitry help us make further progress understanding spacetime? Perhaps. First, maybe such a model can help build a quantum computer that replicates reality. If we can figure out a circuit analog to particle interactions, then we can model the world in a computer. Vice versa, if particle interactions represent circuitry, we can use particle systems to calculate.

A look back

This paper has attempted to organize recent developments in spacetime physics in a comprehensible story line. We're a long way beyond Hubble's observations of the distant nebulae, a long way beyond Rutherford's scattering experiments and the seminal discoveries of quarks and the Standard Model of particle physics. These days, physicists seriously consider bizarre notions like multiple universes, wormholes, the universe as a giant computer. They are trying to decipher the very structure of spacetime itself, what's out there in the void between galaxies. Their models include esoteric mathematics like tensor networks, quantum entanglement, and other ideas based on information theory and quantum computation. These models suggest that Feynman diagrams might be dual to quantum circuitry, i.e. that we might use quantum circuits to model particle interactions. That would fulfill Landauer's requirement that information is physical, and it provides a familiar, plausible physical structure for spacetime.

Future work, most obviously, requires figuring out the various gates that mediate particular particle interactions. Tests include identification of which gates mediate known processes, mimicking those processes in a quantum computer, and predicting new reaction channels resulting from different gate combinations.

Have we solved the two problems posed at the beginning? Have we reconciled quantum field theory and general relativity? Understood the structure of spacetime? Well, we have Maldacena's mathematical model, AdS/CFT, that does both. AdS/CFT has provided remarkable theoretical insights and practical applications. More are undoubtedly on the way. What remains is to figure out whether AdS/CFT is the appropriate guidepost to understanding the real, flat-space universe in which we find ourselves.

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Table 1. Some of the common gates in binary circuits. A few are universal gates: NAND, Toffoli, and Fredkin among them. Complete circuits can be built using just those gates by themselves. Usually, though, and because of limitations in the physical hardware of the computer, various combinations of gates are more efficient. The truth table shows the output for any given binary input.

<table>
<tr><th>Gate</th><th>Truth table</th></tr>
<tr><td>NOT
flips the bit in its wire</td><td><table><tr><td>Input</td><td>Output</td></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table></td></tr>
<tr><td>AND
output 1 if both A and B = 1, output zero otherwise</td><td><table><tr><td>Input AB</td><td>Output C</td></tr><tr><td>00</td><td>0</td></tr><tr><td>01</td><td>0</td></tr><tr><td>10</td><td>0</td></tr><tr><td>11</td><td>1</td></tr></table></td></tr>
<tr><td>NAND (not AND)
output 0 if both A and B = 1, output 1 otherwise</td><td><table><tr><td>Input AB</td><td>Output C</td></tr><tr><td>00</td><td>1</td></tr><tr><td>01</td><td>1</td></tr><tr><td>10</td><td>1</td></tr><tr><td>11</td><td>0</td></tr></table></td></tr>
<tr><td>XOR (exclusive OR)
output 1 if either A or B = 1, output zero otherwise</td><td><table><tr><td>Input AB</td><td>Output C</td></tr><tr><td>00</td><td>0</td></tr><tr><td>01</td><td>1</td></tr><tr><td>10</td><td>1</td></tr><tr><td>11</td><td>0</td></tr></table></td></tr>
</table>

<table>
<tr><td>SWAP
exchanges bits between two wires</td><td><table><tr><th>Input AB</th><th>Output AB</th></tr><tr><td>00</td><td>00</td></tr><tr><td>01</td><td>10</td></tr><tr><td>10</td><td>01</td></tr><tr><td>11</td><td>11</td></tr></table></td></tr>
<tr><td>CNOT
flips target bit, in second wire, if bit in the input wire is 1</td><td><table><tr><th>Input AB</th><th>Output AB</th></tr><tr><td>00</td><td>00</td></tr><tr><td>01</td><td>01</td></tr><tr><td>10</td><td>11</td></tr><tr><td>11</td><td>10</td></tr></table></td></tr>
<tr><td>Toffoli gate
flips bit in target wire C if both input wires, A and B, are 1’s</td><td><table><tr><th>Input ABC</th><th>Output ABC</th></tr><tr><td>000</td><td>000</td></tr><tr><td>001</td><td>001</td></tr><tr><td>010</td><td>010</td></tr><tr><td>011</td><td>011</td></tr><tr><td>100</td><td>100</td></tr><tr><td>101</td><td>101</td></tr><tr><td>110</td><td>111</td></tr><tr><td>111</td><td>110</td></tr></table></td></tr>
<tr><td>Fredkin gate
swaps bits in target wires, B and C, if A is 1</td><td><table><tr><th>Input ABC</th><th>Output ABC</th></tr><tr><td>000</td><td>000</td></tr><tr><td>001</td><td>001</td></tr><tr><td>010</td><td>010</td></tr><tr><td>011</td><td>011</td></tr><tr><td>100</td><td>100</td></tr><tr><td>101</td><td>110</td></tr><tr><td>110</td><td>101</td></tr><tr><td>111</td><td>111</td></tr></table></td></tr>
</table>

Table 2. Standard universal set of quantum gates. Note that these gates are equivalent to vector operators – matrices – that rotate state vectors in three-dimensional vector space. For example, the action of the X gate is to rotate a vector around the X axis. We can choose X to represent direction of a physical parameter such as spin or "direction" in some other state space, such as color charge. For example, an X gate, in matrix form, operating on spin down is represented as

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

where $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is the vector representation of spin down and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is the vector spin up. With this set of gates, we can rotate state vectors to any orientation in space, i.e. we can represent any of the infinitude of states on 2-D or 3-D coordinate systems. See Figure 6.

<table>
<tr><th>Gate</th><th>Truth table or matrix form</th></tr>
<tr><td>CNOT
flips target qubit in the second wire if qubit in the input wire is 1</td><td>
<table>
<tr><td>Input AB</td><td>Output AB</td></tr>
<tr><td>$|00\rangle$</td><td>$|00\rangle$</td></tr>
<tr><td>$|01\rangle$</td><td>$|01\rangle$</td></tr>
<tr><td>$|10\rangle$</td><td>$|11\rangle$</td></tr>
<tr><td>$|11\rangle$</td><td>$|10\rangle$</td></tr>
</table>
</td></tr>
<tr><td>Hadamard
creates mixed states</td><td>
<table>
<tr><td>Input</td><td>Output</td></tr>
<tr><td>$|0\rangle$</td><td>$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$</td></tr>
<tr><td>$|1\rangle$</td><td>$\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$</td></tr>
</table>
</td></tr>
<tr><td>X
rotates state vector around the x-axis</td><td>$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$</td></tr>
<tr><td>Y
rotates state vector around the y-axis</td><td>$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$</td></tr>
</table>

Z rotates state vector around the z-axis	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Phase shift	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\frac{\pi}{8}$ Phase	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix}$