The Quantum Explainer

Here's a brief introduction to key ideas in quantum mechanics. It's meant to help the reader understand new and unfamiliar concepts and to provide a reference for more in-depth study. Hopefully it also provides some fun.

First there's some background to locate quantum mechanics in the larger structure of physics. Next we distinguish the formal mathematical underpinnings of quantum mechanics from the interpretations of those equations; what do they mean? From there, onward to some examples of the quantum weirdness that makes quantum mechanics seem at odds with our everyday experience. Along with the quantum phenomena, we describe applications of quantum mechanics in practical devices and in basic research. Finally, this paper lists some likely resources for further study and suggestions for kitchen experiments that illustrate the ideas.

There's nothing like pictures to help understand abstract ideas. Probably the best set of illustrations, in the form of simulated quantum experiments, can be found at the PhET quantum web site. PhET is the brainchild of Carl Weiman, Nobel Laureate in Physics and gifted educator. PhET offers 'hands on' experiments that simulate real quantum phenomena (an also other realms of science). The simulations are accompanied by experimental guides and instructors notes. Check them out as you read through this explainer.

Quantum mechanics is a physical model.

Physics is the study of the natural world. (By 'world' we mean the whole universe or, these days, multiverse.) Physicists try to understand how the world works. Why the sun shines, why the sky is blue, what happens under the event horizon of a black hole, etc.

In order to understand the world, physicists build mathematical models. A model is an equation that allows you to make a prediction that you can compare with experiment. If the model's prediction agrees with the results of the experiment, then you can assume you've gained some understanding of that particular physical phenomenon.

For example, Isaac Newton provided a model to describe why objects move the way they do. His basic model of motion is

$$F = ma$$

The math is logical shorthand to express ideas. F is the symbol for force. m represents mass, and a is acceleration. The idea is that in order to accelerate a diesel train engine from zero to sixty miles per hour you have to push a whole lot harder (apply a whole lot more force) than if you accelerate a VW beetle. You can test the model by experiment: push on the locomotive, push on the beetle, see if the required forces are proportional to the masses of the two objects.

Same with quantum mechanics. It's a mathematical model. The equations describe how the world works. The model is tested by experiment. Only the concepts in quantum mechanics are more abstract than pushing and pulling on familiar objects.

Quantum mechanics describes what happens at the smallest scales.

Atoms and their constituent parts, that's the realm of quantum mechanics. Tiny distances and short time scales. What keeps electrons from flying off independently from their atomic nuclei. What holds quarks inside of protons. Why atoms absorb and radiate light. That's where the equations of quantum mechanics apply.

Of course, the rest of the world is built from atoms, so we should be able to understand the rest of the world in terms of quantum mechanics. Most physicists think that quantum mechanics is THE underlying model for physics and that eventually we'll be able to frame the other great model, Einstein's general relativity, in terms of quantum mechanics.

Quantum mechanics is two (equivalent) models.

Truth be told, quantum mechanics is really two models, two sets of equations. Turns out they're equivalent; one set of equations can be translated into the other set. That's part of the confusion in learning QM. On the other hand, having two ways of thinking can help. Sometimes the world can be described more easily one way, and sometimes it's easier to calculate using the other model.

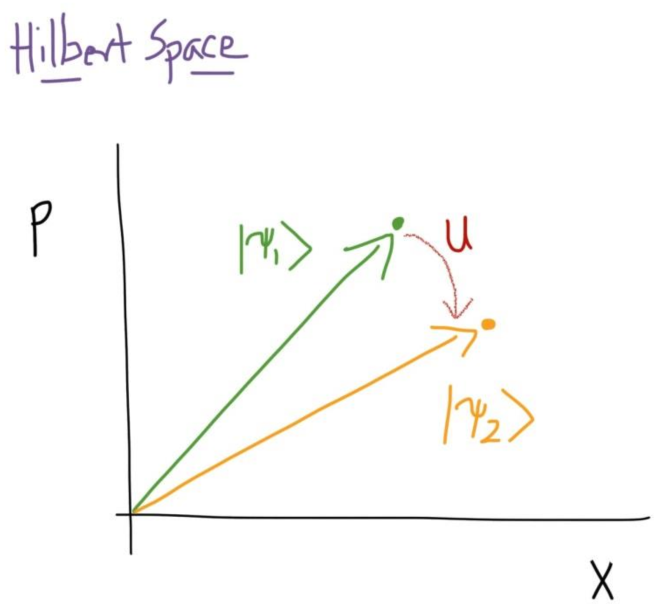
Early in the game (1920's) Erwin Schrodinger developed what he called wave mechanics. His idea was that the subatomic world could be described in the known mathematics of waves. Imagine we're out on the ocean. Waves can be described in terms of their height (amplitude), distance from the crest of one wave to the next (wavelength), and frequency (how many times our boat bounces up and down per second). Schrodinger got the notion that you could use those parameters (frequency, amplitude, wavelength) to model the atomic world. Only difference, what's new in quantum mechanics, was Schrodinger realized that when we measure atomic stuff we're measuring probabilities. What's the probability of finding an electron here instead of over there? What's the probability that this atom will emit a photon of light? And those probabilities are calculated using different rules than the probabilities we learned in high school. (We'll get to the differences shortly.) Schrodinger's model, the Schrodinger equation, is used everywhere and all the time in quantum mechanics.

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$$

Looks beastly, but like Newton's equation it's just symbols to represent ideas. What it's saying, essentially, is that the rate of change of a wavefunction (Greek letter psi, the funny looking pitchfork ψ that represents the wave) is proportional to the energy in the system. Psi might represent an electron or a light wave or a hydrogen atom, depending on what we're trying to figure out.

(Don't let the Greek letters and other strange mathematical symbols scare you. Physicists use them just because they've exhausted the usual Latin alphabet. Newton already grabbed F and m and a, etc., and the mathematicians stole x and y and u and v and so on for their own purposes. The quantum people came late to the game, so they looked to the Greek alphabet to represent their new ideas.)

Well and good for Mr. Schrodinger, but at about the same time Werner Heisenberg invented another representation for the same ideas. Heisenberg's model is referred to as matrix mechanics. While Schrodinger used the mathematical tools of calculus, Heisenberg chose vector algebra. Quantum states are represented as vectors in Hilbert space. (What?!) That is, if you want to describe the position and momentum of an electron, say, you draw an arrow from the origin to the dot that represents its position and momentum. That's called its state vector.



State vectors in Hilbert space: This figure shows an example of a Hilbert space with the state vector representing a particle at two instants of time. $|\psi_2\rangle$ shows the position and momentum of the particle a short time after it was in the state $|\psi_1\rangle$. It has moved to the right and lost some momentum. The change of state was mediated by a time evolution operator *U*, a matrix in the Heisenberg representation that determines how states change over time. Note that we can increase the number of coordinates to include other observables such as spin; just add another axis pointed out of the paper. It's not possible to visualize all the axes for all the observables – a typical Hilbert space would include position axes in three dimensions, momentum along the three dimensions, plus spin and other observables that we might be interested in. We can't see all those dimensions of Hilbert space, but the equations include them readily.

Of course, the electron is always moving, so the state vector changes over time. In vector algebra, the change in the state vector is determined by a matrix operator. For example, to find the state (position and momentum) of the electron at the next instant of time, multiply its state vector by the energy operator (matrix). Or, to use a simpler example, to flip the spin (state vector) of an electron, multiply by the X (spin flip) operator.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Schrodinger's wave mechanics use the powerful analytical tools of calculus. Heisenberg uses algebraic tools that can be implemented readily in computer programs. Freeman Dyson showed that the two approaches are equivalent: different languages to describe the same world.

The observables.

What's that quantum world made of? What are physicists trying to describe? What do we want to calculate with Schrodinger's wavefunctions and Heisenberg's state vectors? Here's a list of the most important observables, those things we can ask questions about and then measure in an experiment. It's these observables that are the key ingredients to understanding the quantum world.

Observable	**symbol**	**description**
position	*x*	where a particle is located at a particular time
momentum	*p*	mass of the particle times its velocity. In everyday terms, the momentum is a measure of how likely it is the object will knock you down. A mosquito traveling at 20 mph won't do much damage. A locomotive traveling at that speed, better get out of the way. In QM, momentum is the rate of change of the wavefunction with position, how fast you're sliding down the wave from one point to another.

energy	*E*	in QM, the rate of change of the wavefunction with time, i.e. how fast is the wave oscillating
spin	*s*	a purely quantum phenomenon, a characteristic of electrons and the other particles. It's called spin because it mimics, in some ways, the properties of a spinning top. It's measured with reference to the axis of spin. Look down on a spinning top. If it's spinning counterclockwise, call it spin up. If it's spinning clockwise, that's spin down.
polarization		photons (beams of light) can be sorted by the polarization of their electric field component. This is how polarized sunglasses work. The light can be polarized horizontally or vertically or at other angles (by superposition)
and a whole bunch of others describing the effects of the strong and weak nuclear forces		

Quantum mechanics vs. the interpretation of quantum mechanics

It's important to keep in mind that the mathematics of quantum mechanics is very well established. Its calculations agree with the results of experiments to remarkable precision. But it is also true that nobody really understands quantum mechanics. It makes some very strange and counter-intuitive predictions (which have been confirmed by experiments). What is an electron, really? What is spin? How can a particle be in two places at the same time? What is the actual reality underlying quantum mechanics? We don't know, but that doesn't stop people from speculating. Here are the favorite guesses. I'll use the 'particle-in-two-places' example to illustrate.

<u>Many-worlds interpretation</u>. A particle can be in two places because it really is in two places at the same time. At every instant of time, the world splits into all the different possibilities where the particle might be. In one universe the particle is here. In another universe it's over there.

<u>Pilot waves</u>. The particles that we measure are carried around by underlying 'probability' waves. Just like ocean waves, the pilot waves spread out all over the place. We can't locate the whole

wave as a definite position. The particle can be riding anywhere on the wave, and we don't know where it is until we measure it.

Copenhagen interpretation. We don't know what's going on in the quantum world until we measure it, so forget about trying to figure out what the world 'really' is. Once we measure that a particle is here and not there, then don't worry about it. It's here. This is the 'shut up and calculate' approach to quantum mechanics. It works, so don't fret about why it works.

Hidden variables. This was Einstein's argument. Even though he helped build the foundations of quantum mechanics, he refused to believe that Nature was probabilistic and non-local (see below). "God doesn't throw dice!" Instead, he argued that quantum mechanics is an incomplete model. Events could be completely predicted, he argued, if we only knew more precisely what was going on inside those quantum particles. There's something inside the system that tells the particle what path it will take in a double slit experiment, for example. We just don't understand that something yet. It's important to note that the Irish physicist John Bell has proved rigorously that there can be no hidden variables. Quantum mechanics really is, at its core, probabilistic. See Bell's book, *Speakable and unspeakable in quantum mechanics*.

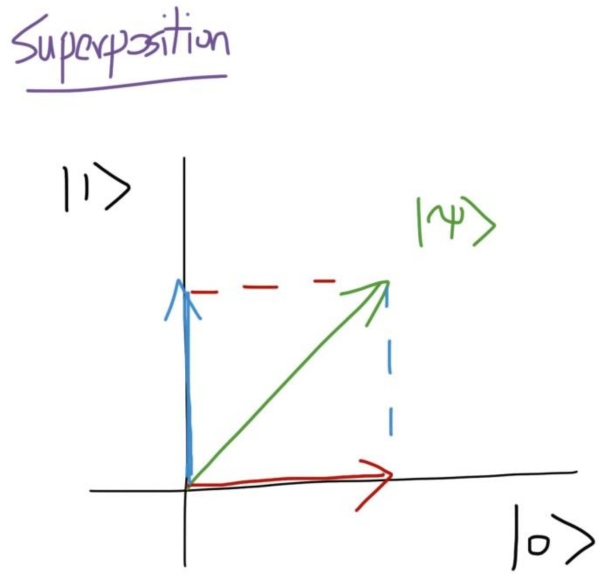
There are other interpretations, but those are the main ones.

The axioms of quantum mechanics

Like all rigorous mathematics, quantum mechanics rests on a set of axioms. They form the foundation on which the model is built. The axioms themselves are supported by experimental evidence. I'll use Schrodinger's term 'wavefunction' in the list of axioms below. Heisenberg would use the term 'state vector' instead.

1. The wavefunction tells all. All the information about a physical system is included in its wavefunction, the ψ in Schrodinger's equation . For example, a free electron scooting through space is described by a particular wavefunction. An electron bound to a hydrogen nucleus is described by another wavefunction.

2. Superposition. It helps in our calculations to think of wavefunctions themselves as superpositions of eigenfunctions. Eigenfunctions are reference vectors that we use to measure the state of a system. Think of eigenfunctions as the coordinate axes familiar from algebra class. They are the references we use to measure everything else on the grid. Pictures help to understand.



The wavefunction / state vector as superposition of two eigenvectors: The state vector $|\psi\rangle$ is the superposition (i.e. vector sum) of two eigenvectors, $|0\rangle$ and $|1\rangle$. The eigenvectors might represent spin up and spin down, or they could represent other observables like two energy states in an atom. The mathematics is general. Main idea is that the result of any measurement on a state $|\psi\rangle$ depends on how we choose to measure, i.e. how we orient our measuring apparatus, the eigenvectors. In the figure, the probability of detecting $|\psi\rangle$ if we measure along the $|0\rangle$ axis is given by the square of the length of the red arrow. The probability of detecting $|\psi\rangle$ if we measure along the $|1\rangle$ axis is given by the square of the length of the blue arrow. See the PhET Stern-Gerlach simulation to get a feel for what's going on. See also the wave superpositions at PhET wave superpositions.

The great physicist Richard Feynman famously said that all quantum mechanics is contained in the double slit experiment. That experiment, done with light or electrons or any other particle, shows the surprising effects of superposition on particle behavior. See PhET wave interference.

3. Time evolution. When no one is looking (measuring), a quantum system evolves according to the time-dependent Schrodinger equation. This axiom tells us that the world hums along merrily when we're not looking. Quantum systems evolve predictably as long as we don't take a peek.

4. Measurement. When we perform a measurement on an evolving quantum system, however, we take a snapshot the captures the probability that a particle happens to be in the state that we are looking for (with our eigenvector measuring tools). What we see depends on how we measure it. For example, if we produce a bunch of electrons with sideways spin, left-to-right, and then we measure spin with an apparatus oriented sideways we will find all the electrons spin sideways, left-to-right. On the other hand, if we measure those sideways electrons with an apparatus oriented up – down, then we'll find half of them spin up, half spin down. By the rules of quantum math, the outcome of a measurement is the square of the amplitude of the component of the wavefunction along the particular eigenbasis (coordinate axis / eigenvector) that we choose for the measurement. See superposition figure above and also PhET Stern-Gerlach.

5. Measurement outcomes are probabilistic. Total probability of finding the particle somewhere is always 1. That is, the particle, assuming it exists, always exists in some state. It's out there somewhere. This is referred to as 'unitarity.' The Stern-Gerlach experiment, linked in 4. above, shows the probabilistic nature of measurement and the collapse of the wavefunction, discussed next.

6. Collapse of the wavefunction. Whenever we measure a quantum system, its wavefunction 'collapses' to one of the eigenstates available in our measuring apparatus. For example, if our measurement finds that the spin of an electron is up, then we will always find it spin up if we measure it again. The quantum system remains in that same state thereafter. Repeated measurements always return the same result.

The weirdness in quantum mechanics

The mathematical axioms of quantum mechanics predict all kinds of strange behavior. This is the stuff everybody talks about, the things that make quantum mechanics so fascinating. Here are a few.

1. Uncertainty. You can't know two things at once about a particle's state. For example, you can't measure its position to arbitrary accuracy and at the same time know its momentum. Nor can you know both its energy and the time interval during which it carried that energy. These are both expressions of Heisenberg's Uncertainty Principle. They apply to other pairs of observables as well. Uncertainty is not just an admission that we can't measure things well. It's built in to the way the world works. There are several weird and wonderful consequences of uncertainty.
   a. Tunneling. Since the position of a particle is uncertain, i.e. its wavefunction is stretched out over space, when the particle encounters a barrier, and if the

barrier's not too wide, there's a chance that the particle can pass straight through the barrier. It's as if you could walk through a wall without using the door. See PhET quantum tunneling.

b. Condensates, superfluids, and superconductivity. Again because wavefunctions spread out, if you collect a bunch of identical particles in a confined region and cool them to really low temperatures their wavefunctions overlap so that the whole bunch of particles behaves like a single entity. This results, for example, in superconductivity: electrons move all together down a wire and without resistance. See Inside Science video on superconductors and also the Bose-Einstein condensate
c. Lasers. These are so commonplace we forget their origin. Lasers produce intense, collimated beams of electromagnetic radiation because their inventors realized a particular class of particles, the bosons, like to bunch together with all their wavefunctions in lock step. Photons, particles of light, are one of the bosons. Challenge was to figure out how to amplify photon production, i.e. tickle atoms just right to produce more and more photons, all of them naturally in tune with all the others. See PhET lasers

2. Entanglement. Produce a pair of entangled particles, i.e. particles with correlated observables. For example, fire a high energy photon to produce a pair of electrons. One will be spin up and the other spin down. That's easy to do in a laboratory. (See Wikipedia references for entanglement.) Give one of the entangled particles to Alice and the other to Bob. (It's always Alice and Bob in these thought experiments.) Alice can carry her particle across the universe or dive into a black hole. Her particle is still entangled with Bob's. If Alice measures her particle and finds its spin is up, then Bob finds his is spin down. No matter how far apart they are, when Alice measures her particle she immediately knows the state of Bob's. Note that this does not violate relativity's caveat that you can't send information faster than the speed of light. Alice knows the state of Bob's particle before he measures it, but Bob won't find out until he receives a message from Alice or measures his particle himself. Applications of entanglement are just beginning. See Quanta Magazine entanglement compendium for a nice collection of articles.
   a. Quantum cryptography. The Chinese already have deployed a prototype quantum communication circuit. The sender and receiver are linked with entangled particles. Any message sent over the circuit is absolutely secure; if anybody is eavesdropping the entanglement is broken and that's immediately detected by the communication system. See can quantum codes really be unbreakable?
   b. The fabric of spacetime. It's been known since the early days of quantum mechanics that 'empty' space is not empty. It seethes with virtual particles popping in and out of the vacuum, a result of the uncertainty relation between

energy and time. In a short enough time interval, nature can borrow energy to create particles, as long as they disappear back into the vacuum within that interval. Nowadays, physicists have arrived at compelling mathematical models showing that spacetime, the vacuum, itself is constructed from entanglement. Zillions of entangled particles scattered across space are all connected, as the thinking goes, by tiny wormhole threads. That's spacetime, the 'void' out there in between the galaxies. It's a web of entanglement. See how quantum pairs stitch spacetime and also how spacetime is built by quantum entanglement

3. Unlocality. The rest of physics, besides quantum mechanics, rests on the assumption that objects that are far apart cannot influence each other without some delay dictated by the speed of light. You can't see the light turn green at an intersection until the light reaches your eye. Because of entanglement, however, a measurement made on one of an entangled pair of particles here on earth immediately determines the outcome of a measurement on its entangled partner in the Andromeda galaxy, 2.5 million light years away. This is one of the puzzles separating quantum mechanics from general relativity. The theory of general relativity absolutely requires the assumption of locality. Trying to resolve this contradiction in the two theories is one of the premier challenges, arguably the great challenge, in current physics.

4. Decoherence. This is the bane of experimentalists and of engineers trying to build quantum computers. It has to do with the collapse of the wavefunction when a quantum system is measured. Problem is that the natural world is always taking measurements on itself. Whenever electron A bumps into electron B, A carries away information about B. A has measured B. Physicists think that this is the reason the world as we see it is not quantum-strange. We don't see the same cat in two places at the same time. There are so many zillions of particles in the cat interacting with so many zillions of particles in its environment that all that information from collapsed wavefunctions is dispersed into the environment. And that's what we see. Read about Wojciech Zurek's work for further insights.

Experiments and demonstrations

You can find a menu of simple experiments at the end of this chapter on quantum mechanics.

One handy set of experimental tools not included in the list of experiments above are polarized filters. Extract the lenses from old polarized sunglasses, get them from a 3D movie, or use the sunglasses that are still resting on your nose. To do all the experiments, you'll need three filters along the path of light. Two lenses will do for the basics. With them you can demonstrate all the main ideas of superposition, quantum measurement, collapse of the wavefunction, Stern-Gerlach, and more. Here's a good start to fun with polarizers.

References

I've listed these in order of getting immersed in quantum mechanics. Dip your toe in with Greene and Gamow. Dive in to the depths with Feynman, Susskind and Griffiths.

Brian Greene. 2011. *Fabric of the Cosmos*. PBS series, provides a wonderful overview of modern physics (which is built largely on QM.) https://www.pbs.org/wgbh/nova/series/the-fabric-of-the-cosmos/

Brian Greene. 2004. *The fabric of the cosmos*. Knopf. The book, with further details and references.

George Gamow. 1985. *Thirty years that shook physics*. Dover. An older book, but good introduction to the main ideas, by one of the participants.

Bob Dorsett. 2011. *Essentials of modern physics*. Chapter 7 is all about quantum mechanics. http://dorsett-edu.us/PhysicsText/PhysicsTextHome.html

Richard Feynman. 2014. *QED: the strange theory of light and matter*. Princeton. Feynman is the physicist's physicist and one of the great teachers (and characters). This book is his presentation of trying to figure out why QM is what it is. You might also be interested in other books about Feynman and his work, e.g. *Surely you're joking, Mr. Feynman*, by Richard Feynman and Ralph Leighton It's great fun.

Tanya and Jeffrey Bub. 2018. *Totally random: why nobody understands quantum mechanics*. Princeton. A nice cartoon exposition, especially of entanglement.

Leonard Susskind and Art Friedman. 2014. *Quantum mechanics: the theoretical minimum*. Basic Books. This is more mathematical, but all QM is here in a nice, tidy package.

David Griffiths. 2004. *Introduction to quantum mechanics*. Pearson. Probably the most accessible of the formal text books, with good explanations in between the math.